

# A new multiresolution near-field to near-field transform suitable for multi-region FDTD schemes.

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**Abstract**— In this paper, new Discrete Wavelet Transform (DWT)-based compression techniques are applied to the computation of the near-field to near-field transform in a Multi-Region FDTD (MR / FDTD) scheme. The compression is applied to both radiating fields on the source surface and incident fields on the excitation surface, which significantly decreases the computation cost of the interactions between the volumes.

## I. INTRODUCTION

FDTD reaches its limitations when computing interactions between remote objects (say a few wavelengths). Multiscale techniques are then required. The Multi-Resolution Time Domain (MRTD) approach introduced by [1] is based on a wavelet expansion of the fields in the computation volume. By using the vanishing moments properties of wavelets, some field components can be neglected, which results in a reduction of the overall computing requirements. However, this technique can be cumbersome when boundary conditions are involved. Moreover, no a priori thresholding criterion exists that permits an automated choice of the wavelet components to neglect.

[2] proposes a different approach relying on a multi-region scheme, hence particularly adapted for the simulation of large, sparsely filled modelling problems. The Multiple Region FDTD (MR / FDTD) associates each element of the problem with its own simulation volume, orientation, meshing steps and boundary conditions, therefore removing the need to mesh the space between the elements. The interactions between the different FDTD volumes are taken into account by means of a near field to near field transform (in our example the Kirchhoff integrals). This technique unfortunately suffers from high computation costs, due to both the number of radiating field components to take into account on the Kirchhoff surface and the number of incident field components to compute on the total field - scattered field

separation surface [3].

A Discrete Wavelet Transform (DWT)-Based compression technique initially proposed to decrease the cost of the near-field to far-field transform [4] has been successfully applied to the Kirchhoff surface [5]. In this paper, we extend this compression technique to the excitation surface in the total field - scattered field formulation. This leads to an important decrease in the total number of field components involved in the near-field to near-field transform.

## II. DISCRETE WAVELET TRANSFORM (DWT)-BASED COMPRESSION

In FDTD, very small mesh sizes are currently used to keep numerical dispersion as low as possible or to give account for small geometrical details in large structures, hence oversampling the fields compared to Nyquist sampling criterion. Expressing the FDTD field components in a wavelet basis provides a very simple though powerful way to remove locally unnecessary information, that has been successfully applied to the computation of near-field to far-field and near-field to near-field transforms [4][5]. The technique is developed below in a Haar basis, for the example of a discrete wavelet transform of order  $n = 1$  applied to four adjacent electric field components  $E_y$  :

$$\begin{bmatrix} E_y^{(i,j)} & E_y^{(i,j+1)} \\ E_y^{(i+1,j)} & E_y^{(i+1,j+1)} \end{bmatrix} = H^{-1} \cdot \frac{1}{2} \begin{bmatrix} E_{\varphi\varphi} & E_{\varphi\psi} \\ E_{\psi\varphi} & E_{\psi\psi} \end{bmatrix} \cdot H \quad (1)$$

with the four  $E_y$  field components expressed, respectively, in the FDTD basis, and in the Haar basis, and  $H$  is the Haar matrix of order  $n = 1$ ,

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad (2)$$

In equation 1,  $E_{\varphi\varphi}$  is the coefficient associated with the Haar scale functions while  $E_{\varphi\psi}$ ,  $E_{\psi\varphi}$  and  $E_{\psi\psi}$  are the coefficients of the wavelets. The proposed compression process consists of neglecting the contribution of the 3 wavelet functions for the incident field

on the total field - scattered field separation surface. In the regular FDTD basis, that gives :

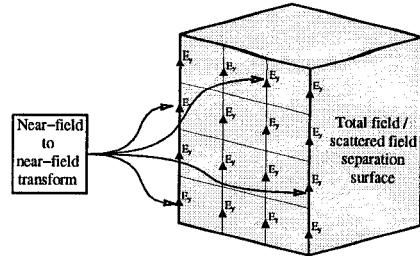
$$E_y^{(i,j)} = E_y^{(i+1,j)} = E_y^{(i,j+1)} = E_y^{(i+1,j+1)} = \frac{E_{\varphi\varphi}}{4} = \bar{E}_y \quad (3)$$

Neglecting the information contained in the wavelets is equivalent to considering the average  $\bar{E}_y$  of the four electric field components  $E_y$  instead of each separate value, whatever the order of decomposition  $n$ .

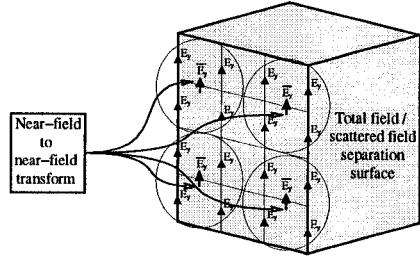
### III. APPLICATION TO THE EXCITATION IN THE TOTAL FIELD - SCATTERED FIELD FORMULATION

The application of this compression technique to the excitation process in the total field - scattered field formulation is straightforward : considering the fact that the source is relatively far from the excitation surface (say a few wavelengths), the spatial variations of the exciting wave can be neglected. In other words, the incident field is only expressed with the scale functions coefficients. However, the resulting diffracted fields would still be expressed in the whole basis. In practice, this means an average excitation is applied to a group of cells, whereas the full FDTD basis is considered for the diffracted fields. In the example shown in figure 1, the computation of 16 incident field  $E_y$  components on a total field - scattered field separation surface is considered. In (a), each  $E_y$  component is computed by means of a near field to near field transform. In (b), our compression algorithm is applied with a compression factor  $2 \times 2$ , i.e. 2 cells in width by 2 cells in height, which means that a near-field component  $\bar{E}_y$  is computed at the center of those 4 near-field components and applied to them, as an average value. The gain in terms of computation time is important, since a compression of  $m$  by  $n$  cells divides by  $m \cdot n$  the number of near-field values to compute, and finally by  $m \cdot n$  the computation time of the interactions between the different MR / FDTD volumes.

The same compression procedure has been previously applied on Kirchhoff surface, i.e. an average field value on  $m'$  by  $n'$  cells of Kirchhoff surface is considered for the surface integrals, decreasing by  $m' \cdot n'$  the computation time of each near-field value [5]. This technique, when applied to the Kirchhoff integrals or to the total field - scattered field excitation formulation, is equivalent to considering coarse meshes for the interactions between the MR / FDTD volumes, while maintaining a thin mesh in each MR / FDTD volume.



(a) Without compression, 16 near-field components to compute.



(b) With compression  $2 \times 2$ , only 4 near-field components to compute.

Fig. 1. Compression applied to the computation of incident  $E_y$  field component on the excitation surface.

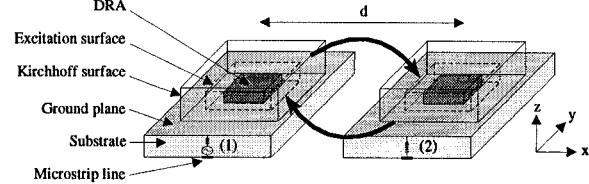


Fig. 2. Computation of the coupling between two DRAs.

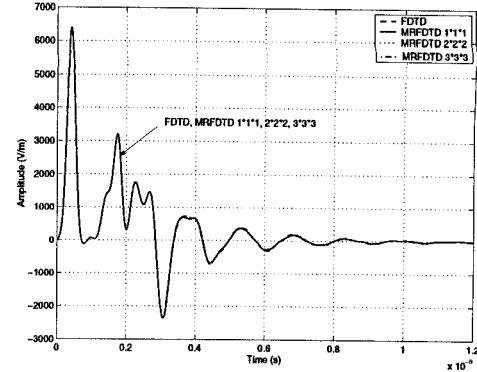
### IV. RESULTS

This compression technique has been applied to the computation of the coupling between two slot fed dielectric resonator antennas (DRA) (see figure 2). The resonance frequency of the DRAs is  $f_0 = 14.3$  GHz, their dimensions being 7 mm by 7 mm (square base) by 2.7 mm height, and the permittivity  $\epsilon_r = 10.2$ . The length and width of the slots are respectively 7.0 mm and 0.5 mm. The excitation is provided through a 2.5 mm wide microstrip line mounted on 0.8 thick duroid substrate ( $\epsilon_r = 2.2$ ). This  $50\Omega$  microstrip line is terminated by a 3.875 mm stub. The excitation is a time gaussian. The spatial meshing steps and the time step are, respectively,  $dx = dy = 0.125$  mm / cell,  $dz = 0.1$  mm / cells, and  $dt = 2.2E^{-13}$  s. The dimensions of the MR / FDTD volumes are

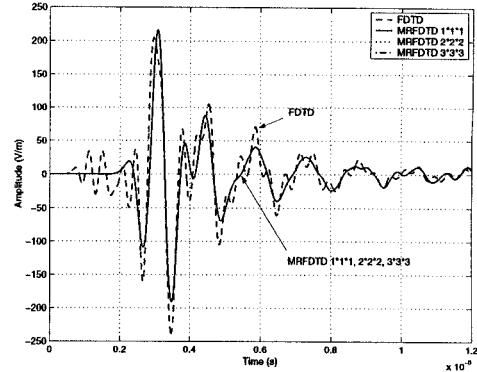
$134 * 159 * 140$  cells. They contain a  $120 * 120 * 60$  cells Kirchhoff surface and a  $64 * 64 * 32$  cells total field - scattered field separation surface. A compression of  $12 * 12 * 12$  cells is applied on Kirchhoff surface, hence already decreasing the computation time of each single near-field value by a factor 144.

For validation purposes, our MR / FDTD code is first compared to FDTD for a distance  $d = 0.6\lambda_0 = 12.5$  mm between the antennas, which is a situation where MR / FDTD would not normally be used, the computation cost of the interactions between the volumes being prohibitive compared to the FDTD volume computation. The dimensions of the FDTD volume are  $234 * 159 * 140$  cells, the observation time being  $T_{obs} = 1.2E^{-9}$  s. Compressions  $1 * 1 * 1$  (i.e. no compression),  $2 * 2 * 2$  (i.e. compression of 2 in width, height and length) and  $3 * 3 * 3$  on the excitation surface are used in MR / FDTD. The obtained signals on voltage fed antenna (1) and matched antenna (2) are plotted in figure 3. FDTD and MR / FDTD show good agreement though some oscillations can be observed on the FDTD signal, due to the discrepancy of our ABCs as well as the propagation from (1) to (2) inside the substrate, which does not occur in MR / FDTD since the volumes are distinct. Note that for a distance  $d = 0.6\lambda_0$  between the DRAs, FDTD outperforms MR / FDTD in terms of computation time (13 hours for FDTD on pentium *III* machine compared to 30 hours for MR / FDTD with compression  $3 * 3 * 3$ ) and memory requirements (227 Mbytes for FDTD versus 267 Mbytes for MR / FDTD). Moreover, MR / FDTD can reach instability for higher compression on the excitation surface, since for close antennas the assumption of low variations of fields on excitation surface does not apply. As a consequence, our compression technique introduces dispersion that is fed to the next volume and fed back to the initial volume, and so on. This confirms the fact that MR / FDTD is not a suitable technique for the simulation of close antennas.

However, this technique quickly outperforms FDTD as antennas get further from each other. As an example, the performances of our compression technique on the excitation surface are evaluated for a distance  $d = 5\lambda_0 = 100$  mm between the DRAs. The dimensions of the FDTD volume are then  $934 * 159 * 140$  cells, the observation time being  $T_{obs} = 2.2E^{-9}$  s. Different values of compression are tested :  $1 * 1 * 1$  (no compression),  $2 * 2 * 2$  (i.e. compression of 2 in width, height and length hence 75% information compression),  $4 * 4 * 4$  (93.75%),  $8 * 8 * 8$  (98.44%),  $16 * 16 * 16$  (99.61%),  $32 * 32 * 32$  (99.90%), and  $64 * 64 * 32$  (only one near-field value computed by face of the total field - scattered field separation



(a) Signal on probe (1).



(b) Signal on probe (2).

Fig. 3. Comparison between FDTD and MR / FDTD without compression ( $1 * 1 * 1$ ) and with compression  $2 * 2 * 2$  and  $3 * 3 * 3$  on excitation surface, for a distance  $d = 0.6\lambda_0$  between the antennas.

surface! (information compression of 99.96%). The signal collected on the matched DRA (probe (2)) is plotted in figure 4, for the different values of compression. Excellent agreement is observed for MR / FDTD with and without compression, for compression factors up to  $32 * 32 * 32$ . (Note that the only compression factor that gives lower quality results is compression factor  $64 * 64 * 32$ , for which there is only one single near-field value computed by face of the excitation surface.) For a distance  $d = 5\lambda_0$  between the antennas, the assumption of low variations of fields on the excitation surface applies and as a consequence, important saving can be achieved in the computation of interactions between MR / FDTD volumes by using the compression technique we propose. High gains in computation time result, as shown in figure 5. MR / FDTD with our compression tech-

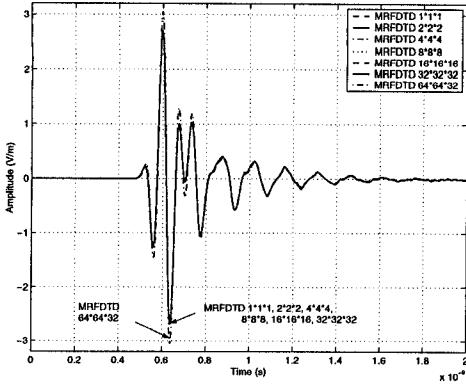


Fig. 4. Signal on probe (2) for MR / FDTD without compression on excitation surface ( $1 \times 1 \times 1$ ), and MR / FDTD with compression  $2 \times 2 \times 2$  up to  $64 \times 64 \times 32$ , for a distance  $d = 5\lambda_0$  between the antennas.

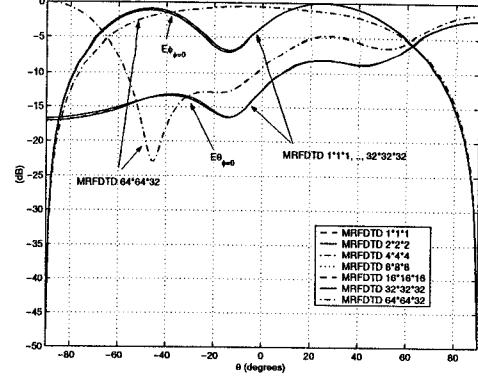


Fig. 6. Far fields re-radiated by antenna (2) with respect to compression on excitation surface, for a distance  $d = 5\lambda_0$  between the source antenna and the illuminated antenna.

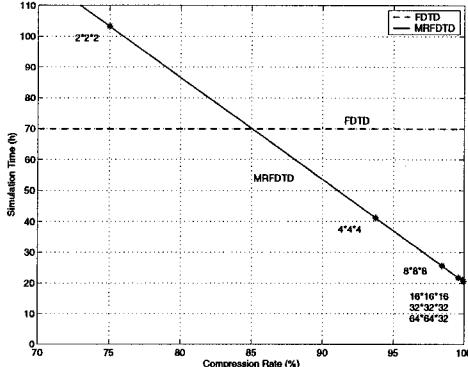


Fig. 5. Computation time for FDTD and MR / FDTD with respect to compression rate, for a distance  $d = 5\lambda_0$  between the antennas.

nique up to compression  $32 \times 32 \times 32$  now outperforms FDTD in computation time without any loss of accuracy (around 21 hours for compressed MR / FDTD against 70 hours for FDTD and 350 hours for MR / FDTD without compression!). The  $x$ -plane far-fields re-radiated by antenna (2) are shown in figure 6 with respect to compression on the excitation surface. Again, very good agreement is observed for MR / FDTD with different compression factors on the excitation surface up to  $32 \times 32 \times 32$ , hence validating our technique for re-radiated far fields.

## V. CONCLUSION

New DWT-based compression techniques have been presented for the computation of near-field to near-field transform in a Multi-region FDTD scheme. Results show high gains in terms memory require-

ments and computation time for MR / FDTD over FDTD as soon as antennas using Kirchhoff transform. Thanks to this technique, it has been shown that the computation time of the interactions between the MR / FDTD volumes can be significantly reduced. This technique appears as a very efficient and very simple way of using wavelets in time domain schemes. In fact, it consists of using a fine mesh when computing local interactions and a coarser one when considering remote couplings.

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## REFERENCES

- [1] M. Krumpholz, L. P. B. Katehi, "MRTD: New Time-Domain Schemes Based on a Multiresolution Analysis", IEEE Transactions on Microwave Theory and Techniques, Vol. 44, no. 4, pp. 555-571, April 1996.
- [2] J. Michael Johnson, Yahya Rahmat-Samii, "MR / FDTD: A Multiple Region Finite Difference Time Domain Method", Microwave and Optical Technology Letters, Vol. 14, no. 2, pp. 101-105, February 1997.
- [3] A. Taflove, "Computational Electrodynamics. The Finite Difference Time Domain Method", Artech House, Norwood, pp. 111-136, 1995.
- [4] G. Carat, R. Gillard, J. Citerne, J. Wiart, "An efficient analysis of planar microwave circuits using a DWT Based Haar MRTD scheme", IEEE Transactions on Microwave Theory and Techniques, Vol. 48, no. 12, December 2000.
- [5] A. Laisné, R. Gillard, J. Citerne, G. Piton, "A Discrete Wavelet Transform (DWT)-Based Compression Technique for the Computation of Kirchhoff Integrals in MR / FDTD", Microwave and Optical Technology Letters, Vol. 27, no. 5, pp. 312-316, December 2000.